

Electromagnetic source for the Kerr-Newman geometry

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Abstract

Source-free equations of nonlinear electrodynamics minimally coupled to gravity admit regular axially symmetric asymptotically Kerr-Newman solutions which describe electrically charged rotating black holes and spinning solitons. Asymptotic analysis of solutions shows the existence of de Sitter vacuum interior which has the properties of a perfect conductor and ideal diamagnetic. The Kerr ring singularity (a naked singularity in the case without horizons) is replaced with a superconducting current which serves as a non-dissipative source of the Kerr-Newman fields and can be responsible for an unlimited life time of a spinning object.

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The Kerr-Newman solution to the Maxwell-Einstein equations was obtained in 1965 [1]

$$ds^2 = \left(\frac{2mr - e^2}{\Sigma} - 1 \right) dt^2 + \frac{\Sigma}{\Delta} dr^2 - \frac{2a(2mr - e^2) \sin^2 \theta}{\Sigma} dt d\phi \\ + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{(2mr - e^2)a^2 \sin^2 \theta}{\Sigma} \right) \sin^2 \theta d\phi^2 \quad (1)$$

with using the Newman-Janis algorithm [2]. Here

$$\Sigma = r^2 + a^2 \cos^2 \theta; \quad \Delta = r^2 - 2mr + a^2 + e^2. \quad (2)$$

The associated electromagnetic potential is given by [1] $A_\mu = -(er/\Sigma)[1; 0, 0, -a \sin^2 \theta]$.

In 1968 Carter found that the parameter a couples with the mass m to give the angular momentum $J = ma$ and independently couples with the charge e to give an asymptotic magnetic dipole moment $\mu = ea$, so that the gyromagnetic ratio e/m is exactly the same as predicted for a spinning particle by the Dirac equation [3].

Carter discovered also that in the case appropriate for a particle, $a^2 + e^2 > m^2$, when there are no Killing horizons and the manifold is geodesically complete (except for geodesics which reach the singularity), any point can be connected to any other point by both a future and a past directed time-like curve which originate in the region where $g_{\phi\phi} < 0$, can extend over the whole manifold and cannot be removed by taking a covering space [3].

The source models for the Kerr-Newman fields, involving a screening or covering of causally dangerous region, can be divided into disk-like [4, 5, 6, 7], shell-like [8, 9, 10, 11], bag-like [12, 13, 14, 15, 16, 17], and string-like ([18, 19] and references therein). The problem of matching the Kerr-Newman exterior to a rotating material source does not have a unique solution, since one is free to choose arbitrarily the boundary between the exterior and the interior [4] as well as an interior model.

The problem of a regular source for the Kerr-Newman electromagnetic fields can be approached in the frame of nonlinear electrodynamics coupled to gravity (NED-GR)¹.

Nonlinear electrodynamics was proposed by Born and Infeld in 1934 as founded on two basic points: to consider electromagnetic field and particles within the frame of one physical entity which is electromagnetic field; to avoid letting physical quantities become infinite [23]. In their theory particles are considered as singularities of the field, with mass of electromagnetic origin, but it is also possible to obtain the finite electron radius by introducing an upper limit on the electric field. In both cases a total energy is finite [23].

The Born-Infeld program can be realized in nonlinear electrodynamics minimally coupled to gravity. Source-free NED-GR equations admit regular causally safe axially symmetric asymptotically Kerr-Newman solutions [24], which describe regular electrically charged rotating black holes and spinning solitons (more precisely the Coleman lumps which are non-singular non-dissipative objects holding themselves together by self-interaction [25]).

The key point is that for any gauge-invariant Lagrangian $\mathcal{L}(F)$, stress-energy tensor of electromagnetic field

$$\kappa T_\nu^\mu = -2\mathcal{L}_F F_{\nu\alpha} F^{\mu\alpha} + \frac{1}{2}\delta_\nu^\mu \mathcal{L}; \quad \kappa = 8\pi G; \quad \mu, \nu = 0, 1, 2, 3 \quad (3)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\mathcal{L}_F = d\mathcal{L}/dF$, in the spherically symmetric case has the algebraic structure

$$T_t^t = T_r^r \quad (p_r = -\rho). \quad (4)$$

Regular spherically symmetric solutions with stress-energy tensors of any origin specified by (4) and satisfying the weak energy condition (non-negativity of density as measured by any local observe), have obligatory de Sitter center with $p = -\rho$ [26, 27, 28] where $p = p_r = p_\perp$ and $p_\perp = -\rho - r\rho'/2$. In NED-GR regular solutions interior de Sitter vacuum provides a proper cut-off on self-interaction divergent for a point charge [30, 24].

Nonlinear electrodynamics minimally coupled to gravity is described by the action

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [R - \mathcal{L}(F)]; \quad F = F_{\mu\nu} F^{\mu\nu}, \quad (5)$$

where R is the scalar curvature. The Lagrangian $\mathcal{L}(F)$ should have the Maxwell limit, $\mathcal{L} \rightarrow F$, $\mathcal{L}_F \rightarrow 1$ in the weak field regime. Variation with respect to A^μ and $g_{\mu\nu}$ yields the dynamical equations

$$\nabla_\mu (\mathcal{L}_F F^{\mu\nu}) = 0; \quad \nabla_\mu {}^*F^{\mu\nu} = 0. \quad (6)$$

where ${}^*F^{\mu\nu} = \eta^{\mu\nu\alpha\beta} F_{\alpha\beta}/2$; $\eta^{0123} = -1/\sqrt{-g}$, and the Einstein equations $G_{\mu\nu} = -\kappa T_{\mu\nu}$ with an electromagnetic source $T_{\mu\nu}$ given by (3).

NED-GR equations do not admit regular spherically symmetric solutions with the Maxwell center [31], but they admit regular solutions with the de Sitter center [30]. The question of correct description of NED-GR regular electrically charged structures in the frame of the Lagrange dynamics is clarified in [32].

Regular spherically symmetric solutions satisfying (4) are described by the metric

$$ds^2 = g(r)dt^2 - \frac{dr^2}{g(r)} - r^2 d\Omega^2; \quad g(r) = 1 - \frac{2\mathcal{M}(r)}{r}, \quad (7)$$

¹NED theories appear as low-energy effective limits in certain models of string/M-theories [20, 21, 22].

where $\mathcal{M}(r) = 4\pi \int_0^r \tilde{\rho}(x)x^2 dx$ is calculated with the electromagnetic density $\tilde{\rho}(r) = T_t^t(r)$ from (3). The metric (7) has the de Sitter asymptotic as $r \rightarrow 0$ and the Reissner-Nordström asymptotic as $r \rightarrow \infty$ [30].

The regular spherical solutions generated by (4) belong to the Kerr-Schild class [33, 16, 34] and can be transformed by the Gürses-Gürsey algorithm [35] into regular axially symmetric solutions which describe regular rotating electrically charged objects, asymptotically Kerr-Newman for a distant observer [36, 24].

In the Boyer-Lindquist coordinates the rotating metric reads [35]

$$ds^2 = \frac{2f(r) - \Sigma}{\Sigma} dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 - \frac{4af(r) \sin^2 \theta}{\Sigma} dt d\phi \\ + + \left(r^2 + a^2 + \frac{2f(r)a^2 \sin^2 \theta}{\Sigma} \right) \sin^2 \theta d\phi^2; \quad \Sigma^2 = r^2 + a^2 \cos^2 \theta, \quad (8)$$

where $\Delta = r^2 + a^2 - 2f(r)$. A function $f(r) = r\mathcal{M}(r)$ comes from a spherically symmetric solution [35]. Regular spherical NED-GR solutions satisfy the weak energy condition, since $\kappa(p_\perp + \rho) = -F\mathcal{L}_F$, $F \leq 0$ and $\mathcal{L}_F \geq 0$ [30]. In consequence, $\mathcal{M}(r)$ is non-negative function growing from $4\pi\tilde{\rho}(0)r^3/3$ as $r \rightarrow 0$ to $m - e^2/2r$ as $r \rightarrow \infty$ [30]. This guarantees the causal safety on the whole manifold due to $f(r) \geq 0$ and $g_{\phi\phi} > 0$ in (8).

The coordinate r is defined as an affine parameter along either of two principal null congruences, and the surfaces of constant r are the oblate confocal ellipsoids

$$r^4 - r^2(x^2 + y^2 + z^2 - a^2) - a^2 z^2 = 0, \quad (9)$$

which degenerate, for $r = 0$, to the equatorial disk

$$x^2 + y^2 \leq a^2, \quad z = 0, \quad (10)$$

centered on the symmetry axis and bounded by the ring $x^2 + y^2 = a^2$ ($r = 0, \theta = \pi/2$) [37].

Rotation transforms the de Sitter center to the de Sitter disk (10). In the co-rotating frame [16] the eigenvalues of the stress-energy tensor are related to the function $f(r)$ as $\kappa\Sigma^2\rho = 2(f'r - f)$; $\kappa\Sigma^2 p_\perp = 2(f'r - f) - f''\Sigma$ [16]. This gives

$$\kappa\rho(r, \theta) = \frac{r^4}{\Sigma^2} \tilde{\rho}(r); \quad \kappa(p_\perp + \rho) = 2 \left(\frac{r^4}{\Sigma^2} - \frac{r^2}{\Sigma} \right) \tilde{\rho}(r) - \frac{r^3}{2\Sigma} \tilde{\rho}'(r), \quad (13)$$

where $\tilde{\rho}(r)$ is a relevant spherically symmetric density profile. In the limit $r \rightarrow 0$, on the disk (10), $r^2/\Sigma \rightarrow 1$ [24]. For the spherical solutions regularity requires $r\tilde{\rho}'(r) \rightarrow 0$ as $r \rightarrow 0$ [30]. As a result we obtain on the disk the equation of state

$$p_\perp + \rho = 0 \quad \rightarrow \quad p_\perp = p_r = -\rho, \quad (14)$$

which represents the rotating de Sitter vacuum [24].

In terms of the 3-vectors of the electric induction \vec{D} and magnetic induction \vec{B} , defined as

$$E_j = \{F_{j0}\}, D^j = \{\mathcal{L}_F F^{0j}\}, B^j = \{^* F^{j0}\}, H_j = \{\mathcal{L}_F ^* F_{0j}\}, \quad (15)$$

where $j, k = 1, 2, 3$, the field equations (6) take the form of the Maxwell equations. The inductions \vec{D} and \vec{B} are connected with the electric and magnetic field intensities by

$$D^j = \epsilon_k^j E^k; \quad B^j = \mu_k^j H^k, \quad (16)$$

where ϵ_j^k and μ_j^k are the tensors of the electric and magnetic permeability [24]

$$\epsilon_r^r = \frac{(r^2 + a^2)}{\Delta} \mathcal{L}_F; \epsilon_\theta^\theta = \mathcal{L}_F; \mu_r^r = \frac{(r^2 + a^2)}{\Delta \mathcal{L}_F}; \mu_\theta^\theta = \frac{1}{\mathcal{L}_F}. \quad (17)$$

The dynamical equations (6) are satisfied by [24, 38]

$$\begin{aligned} F_{10} &= \frac{q}{\Sigma^2 \mathcal{L}_F} (r^2 - a^2 \cos^2 \theta); \quad F_{02} = \frac{q}{\Sigma^2 \mathcal{L}_F} a^2 r \sin 2\theta; \\ F_{31} &= a \sin^2 \theta F_{10}; \quad a F_{23} = (r^2 + a^2) F_{02} \end{aligned} \quad (18)$$

in the limit $\mathcal{L}_F \rightarrow \infty$, and in the weak field limit $\mathcal{L}_F = 1$ where they coincide with the Kerr-Newman fields [3, 14] and an integration constant q is identified as the electric charge.

The relation connecting density and pressure with the electromagnetic fields reads [24]

$$\kappa(p_\perp + \rho) = 2\mathcal{L}_F \left(F_{10}^2 + \frac{F_{20}^2}{a^2 \sin^2 \theta} \right). \quad (19)$$

The field functions (18) give on the disk [24, 38]

$$\mathcal{L}_F \Sigma^2 = \frac{2q^2}{(p_\perp + \rho)}; \quad F = -\frac{\kappa^2 (p_\perp + \rho)^2 \Sigma^2}{2q^2}. \quad (20)$$

Equation of state on the disk (14) requires $\mathcal{L}_F \Sigma^2 \rightarrow \infty$ and hence $\mathcal{L}_F \rightarrow \infty$ faster than Σ^{-2} . The electric permeability in (17) goes to infinity, the magnetic permeability vanishes, so that the de Sitter vacuum disk has both perfect conductor and ideal diamagnetic properties. The magnetic induction \mathbf{B} vanishes on the disk [24].

In electrodynamics of continued media the transition to a superconducting state corresponds to the limits $\mathbf{B} \rightarrow 0$ and $\mu \rightarrow 0$ in a surface current $\mathbf{j}_s = \frac{(1-\mu)}{4\pi\mu} [\mathbf{nB}]$, where \mathbf{n} is the normal to the surface. The right-hand side then becomes indeterminate, and there is no condition which would restrict the possible values of the current [39]. On the de Sitter disk we can apply definition of a surface current for a charged surface layer, $4\pi j_k = [e_{(k)}^\alpha F_{\alpha\beta} n^\beta]$ [4], where $[..]$ denotes a jump across the layer; $e_{(k)}^\alpha$ are the tangential base vectors associated with the intrinsic coordinates on the disk t, ϕ , $0 \leq \xi \leq \pi/2$ labeled by $k=1,2,3$; $n_\alpha = (1 + q^2/a^2)^{-1/2} \cos \xi \delta_\alpha^1$ is the unit normal directed upwards [4]. With using asymptotic solutions (18) and magnetic permeability from (17), $\mu_r^r = \mu_\theta^\theta = \mu = 1/\mathcal{L}_F \rightarrow 0$, we obtain the surface current

$$j_\phi = -\frac{q}{2\pi a} \sqrt{1 + q^2/a^2} \sin^2 \xi \frac{\mu}{\cos^3 \xi}. \quad (21)$$

At approaching the ring $r = 0$, $\xi = \pi/2$, both terms in the second fraction go to zero independently. As a result the surface currents on the ring can be any and amount to a non-zero total value.

In terms of a density $\tilde{\rho}(r)$ and pressure of a related spherical solution, $p_\perp = -\tilde{\rho} - r\tilde{\rho}'/2$ [30],

$$\kappa(p_\perp + \rho) = \frac{r|\tilde{\rho}'|}{2\Sigma^2} \mathcal{E}(r, z) = (r^4 - z^2 P(r)), P(r) = \frac{2a^2}{r|\tilde{\rho}'|} (\tilde{\rho} - p_\perp). \quad (22)$$

This implies a possibility of generic violation of the weak energy condition (WEC) which was reported for several regular rotating solutions [16, 40, 41, 42]. WEC can be violated beyond

a de Sitter vacuum surface $\mathcal{E}(r, z) = 0$ defined by $p_\perp + \rho = 0$ where the right-hand side in (22) can change sign provided that the dominant energy condition ($\tilde{\rho} \geq \tilde{p}_k$) is valid for related spherical solutions.

Each point of the \mathcal{E} -surface belongs to some of confocal ellipsoids (9) covering the whole space as the coordinate surfaces $r = \text{const}$. In the Cartesian coordinates, $x^2 + y^2 = (r^2 + a^2) \sin^2 \theta$; $z = r \cos \theta$, the squared width of the \mathcal{E} -surface, $W_\mathcal{E}^2 = (x^2 + y^2)_\mathcal{E} = (a^2 + r^2)(1 - z^2/r^2)_\mathcal{E} = (a^2 + r^2)(1 - r^2/P(r))$. For regular solutions $r\tilde{\rho}' \rightarrow 0$, $p_\perp \rightarrow -\rho$ as $r \rightarrow 0$ [30], and $P(r) \rightarrow A^2 r^{-(n+1)}$ with the integer $n \geq 0$ as $r \rightarrow 0$. Then $W_\mathcal{E}^2 = (a^2 + r^2)(1 - r^{(n+3)}/A^2)$, and $W^2 \rightarrow a^2$ when $r \rightarrow 0$. As a function of z , $W_\mathcal{E}^2 = (x^2 + y^2)_\mathcal{E} = (a^2 + |z|\sqrt{P(r)})(1 - |z|/\sqrt{P(r)})$, and \mathcal{E} -surface is entirely confined within the r_* -ellipsoid whose minor axis coincides with $|z|_{\max}$ for the \mathcal{E} -surface [38]. The derivative of $W_\mathcal{E}(z)$ near $z \rightarrow 0$ behaves as $z^{-(n+1)/(n+5)}$ and goes to $\pm\infty$ as $z \rightarrow 0$, so that the function $W_\mathcal{E}(z)$ has the cusp at approaching the disk and two symmetric maxima between $z = \pm r_*$ and $z = 0$ [38].

In Fig.1 [38] \mathcal{E} - surface is plotted for the regularized Coulomb profile [30]

$$\tilde{\rho} = \frac{q^2}{(r^2 + r_q^2)^2}; \quad r_q = \frac{\pi q^2}{8m}. \quad (23)$$

Its width in the equatorial plane $W_\mathcal{E} = a$ and the height $H_\mathcal{E} = |z|_{\max} = \sqrt{ar_q}$. Its precise form depends on the relation of two parameters, $\alpha = a/m$ and $\beta = q/m$. For black holes the parameter β changes within the range $0 < \beta < 0.99$ [43, 44]. For $\alpha < (\pi/8)\beta^2$ the \mathcal{E} -surface is prolate. It can be the case for a slowly rotating moderately charged black hole. For $\alpha > (\pi/8)\beta^2$, the \mathcal{E} -surface, shown in Fig.1, is oblate. It can be the case of slightly charged rotating black holes and extreme black holes. It is also the case of electromagnetic soliton (e-lump).

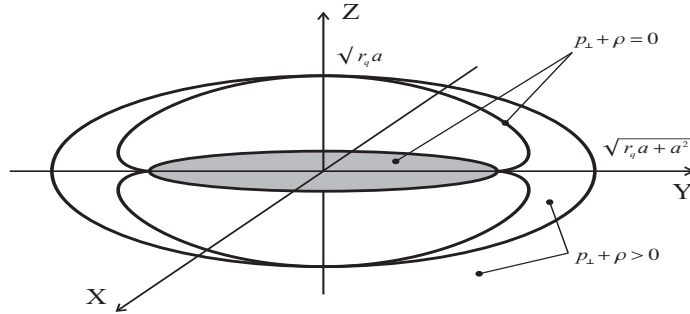


Figure 1: \mathcal{E} -surface for the case $\alpha > (\pi/8)\beta^2$.

\mathcal{E} -surface is defined by $p_\perp + \rho = 0$, it follows $\mathcal{L}_F \rightarrow \infty$ by virtue of the first expression in (20). The magnetic permeability vanishes and electric permeability goes to infinity, so that \mathcal{E} -surface displays the properties of a perfect conductor and ideal diamagnetic.

The de Sitter vacuum \mathcal{E} -surface contains de Sitter disk as a bridge. Magnetic induction vanishes throughout the whole surface by virtue of (15) with taking into account (17) and (18). Within \mathcal{E} -surface, in cavities between its upper and down boundaries and the bridge, a negative value of $(p_\perp + \rho)$ in (19) would mean negative values for electric and magnetic permeabilities in (17) inadmissible in electrodynamics of continued media [39].

The alternative compatible with regularity is zero value of $(p_\perp + \rho)$ also inside \mathcal{E} -surface. This could be the case for the shell-like models ([11] and references therein) and baglike models

([17] and references therein) with the flat vacuum interior, zero interior fields and in consequence zero density and pressures.

The other possibility, favored by the underlying idea of nonlinearity replacing a singularity and suggested by vanishing of magnetic induction on the surrounding \mathcal{E} -surface, is extension of $\mathcal{L}_F \rightarrow \infty$ to its interiors. This results in $p_\perp = -\rho$ and we have de Sitter vacuum core with the properties of a perfect conductor and ideal diamagnetic and vanishing magnetic induction.

The \mathcal{E} -surface exists in the case when a related spherical solution satisfies the dominant energy condition. In the opposite case $p_\perp + \rho \geq 0$ throughout the whole manifold. The de Sitter vacuum disk $p_\perp + \rho = 0$, confined by the de Sitter ring replacing the Kerr singularity, exists for any regular rotating object.

In any case superconducting currents flowing on the de Sitter vacuum ring can be considered as a source of the Kerr-Newman electromagnetic fields. This kind of a source is non-dissipative so that life time of a spinning electrically charged object, in particular of the electron, can be practically unlimited.

For the electron $q = -e$, $ma = \hbar/2$ [3], $a = \lambda_e/2$, where $\lambda_e = \hbar/(m_e c)$ is the Compton wavelength. In the observer region $r \gg \lambda_e$

$$E_r = -\frac{e}{r^2} \left(1 - \frac{\hbar^2}{m_e^2 c^2} \frac{3 \cos^2 \theta}{4r^2} \right); \quad E_\theta = \frac{e\hbar^2}{m_e^2 c^2} \frac{\sin 2\theta}{4r^3}; \quad (24a)$$

$$B^r = -\frac{e\hbar}{m_e c} \frac{\cos \theta}{r^3} = 2\mu_e \frac{\cos \theta}{r^3}; \quad B_\theta = -\mu_e \frac{\sin \theta}{r^4}. \quad (24b)$$

The Planck constant appears here due to discovered by Carter ability of the Kerr-Newman solution to present the electron as seen by a distant observer. In terms of the Coleman lump eq. (24) describes the following situation: The leading term in E_r gives the Coulomb law as the classical limit $\hbar = 0$, the higher terms represent the quantum corrections.

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